

# Tool Trajectory Generation Based on Tool Deflection Effects in the Flat-End Milling Process ( II ) -Prediction and Compensation of Milled Surface Errors-

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In this paper, we present the optimum reference for the compensation of tool trajectory to fulfill the imposed tolerances in flat-end milling process. First, we suggest the milled surface prediction methods considering the tool deflection effects. Based on the predicted error distributions, we propose a cutting process simulation method, which can verify (1) if the tolerance is fulfilled, and (2) if it is possible to compensate the surface errors by the proposed approaches presented in Part I, before the real milling operation begins. As the result of our research, we could make our compensation method very applicable for real industrial cases by considering the imposed manufacturing tolerances. Required experiments for practical examples were performed, and it was found that the results were in good agreement with our predictions.

**Key Words :** CAD/CAM, Flat-End Milling, Tool Path Compensation, Tool Deflection, Surface Error Prediction, Manufacturing Tolerance

## 1. Introduction

In Part I (Seo & Cho, 1999) of this paper, we proposed a surface error compensation strategy based on the prediction of cutting forces and tool deflection. In Part I, the deflection amount at the bottom of the tool was used as the compensation reference. However, if more precise compensation results are needed, this simple reference is not sufficient to investigate the variation of tool deflection along the tool axis. Moreover, the cutting forces vary with the angular position of the tool (Tlustý & McNeil, 1975; Armarego & Deshpande, 1991) for each revolution. The tool deflection also varies with respect to the cutting forces. On the other hand, the shape of the milled surface does not exactly correspond to the deflected end mill, but it can be determined by investigating the interaction between the tool flute and the wor-

piece. Thus, to predict the surface errors exactly, it is necessary to analyze the deflection behavior of the tool in the cutting process. We propose a surface error model, which takes into account the significant contact points between the tool flute and the workpiece. The contact points play predominant roles in generating the milled surfaces.

When using a flat-end cutter in the cutting process, it is apparent that the largest deflection occurs at the bottom of the tool. Thus, the compensation has been carried out in order to minimize the deflection at the bottom. However, the surface errors cannot be compensated appropriately at other positions since the tool deflection varies along the tool axis. It is therefore necessary to choose a reasonable reference for global minimization of all errors in the compensation procedure. In this paper, we have tried to determine the compensation reference by characterizing the significant errors to satisfy the imposed machining tolerance.

Based on the surface prediction, we have established a cutting process simulation method. This simulation method allows us to verify two important points. The first point is to verify whether or

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not the imposed tolerance is satisfied. For given cutting conditions, it is necessary to forecast if the surface errors will be within the imposed tolerance interval. When the tolerance is satisfied, the compensation is not needed. Contrarily, when it is not satisfied, it is necessary to compensate the tool deflection. In this case, it is also necessary to predict the feasibility of the tool path compensation. If the errors are not in the tolerance interval after compensation, the tool path compensation procedure has no meaning. Thus, appropriate verification steps are required to avoid this meaningless compensation before the actual milling operation. The proposed cutting process simulation is employed to predetermine a reasonable solution for current cutting conditions.

To verify the proposed approach, we treat several practical examples. By comparing the simulation results with the experimental results, we show the advantages of the proposed tool path compensation strategy.

## 2. Error Analysis by Predicting the Milled Surface

The main objective of this research is not to reduce the tool deflection itself, but to compensate the surface errors (caused by tool deflections) by adjusting the tool path appropriately. Thus, it is an essential step to estimate the tool deflection effects on the milled surfaces. Among other primary factors leading to surface errors, cusps due to cutter rotation and translation can also be taken into account. In an ideal cutting process, cusp height is represented by a function of tool diameter  $D_T$  and feedrate  $F_D$  (Martellotti, 1941; Martellotti, 1945). However, in our case ( $D_T=6\text{mm}$ ,  $F_D=0.02\text{mm/tooth}$ ), the surface errors due to the cusp height can be negligible compared with the tool deflection. We present here a surface prediction model by considering the deflection effects as predominant factors.

It is known that the shape of the milled surface does not exactly correspond to that of the deformed tool. This mismatch has two causes. First, the surfaces are generated by a twisted cutting edge with a helix angle. Second, in spite of the

same cutting conditions, the cutting forces vary according to the cutter rotation angle in the milling operation (Kline et al., 1982; Sutherland & DeVor, 1986; Tlustý et al., 1991). Thus, these causes have to be integrated systematically in order to obtain satisfactory surface prediction results. We take into account the significant contact points between the cutting edge and the workpiece, which play the most important role in generating the milled surfaces.

### 2.1 Prediction of the milled surfaces

Since the milled surfaces result from the traces on the workpiece generated by the cutter, the movement of the cutting edge should be investigated in order to predict the surface form precisely. It depends on machining behaviors of the cutting tool (Kang, 1996). In ideal milling operations, rotations of the cutter edge make trochoidal form traces, but they interfere with each other. To determine the final trace shapes remaining on the surface, it is necessary to take into account a particular point on the cutter edge. This is an intersection point between the cutter edge and the workpiece, called the Contact Point. Actually, there are an infinite number of contact points along the cutter edge. Among them, we are interested in the most significant contact point, which contributes to generate the final milled surface. The significant contact point is defined as an intersection point between the cutter flute and a vertical plane in the feed direction, which contains the normal vector  $V_N$  and the tool axis (cf. Fig. 1). This fact is based on the assumption that all other traces are removed by following the tool

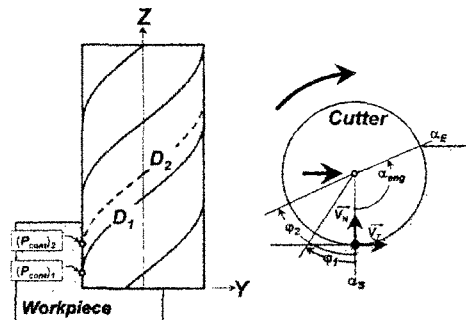


Fig. 1 Significant contact point and cutter edge

rotation. Therefore, only the traces generated by the significant contact point, as defined above, will remain on the surface after the passing of the tool. Figure 1 geometrically illustrates the significant contact point.

The projections of the tool flutes on the lateral surface (plane YZ) can be represented by sinusoidal curves (Fujii et al., 1979). The equation of the projected tool flute is given as

$$Y_n = R \sin \left[ \frac{2\pi}{T} \left\{ z - \frac{(n-1)T}{N_p} \right\} \right], \quad n=1, 2, \dots, N_p, \quad (1)$$

where  $Y_n$  is the  $n^{\text{th}}$  projected profile of the tool flute;  $T$  is the period of this sinusoidal function equal to the lead of a helical tool flute;  $R$  is the tool radius; and  $N_p$  is the number of tool teeth. Eq. (1) corresponds to  $N_p$  sinusoidal curves having a phase difference of  $T/N_p$ . Setting the helical angle of the tool as  $\alpha_{H1}$ , we can obtain

$$T = \frac{2\pi R}{\tan \alpha_{H1}} \quad (2)$$

As shown in Fig. 1, the curves  $D_i$  ( $i=1, 2, 3$  and 4) represent four cutter edges, and  $(P_{\text{cont}})_1$  is the contact point between curve  $D_1$  and the workpiece. As curve  $D_1$  evolves toward curve  $D_2$ , contact point  $(P_{\text{cont}})_1$  moves to  $(P_{\text{cont}})_2$ , since the cutting tool rotates and advances. Hence, the contact point reaches a straight line  $\{(P_{\text{cont}})_1, (P_{\text{cont}})_2\}$  in the plane YZ. However, it is not perpendicular from other viewpoints, due to the cutter translation along the tool path.

Figure 2 schematizes the comportment of the contact points generated by a cutter at two consecutive positions (indicated by Tool-1 and Tool

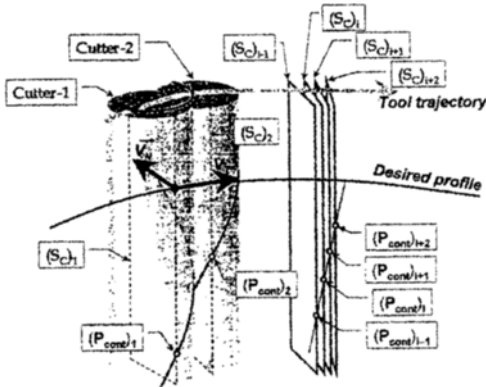


Fig. 2 Geometrical definition of contact points

-2) from another perspective. These tool positions have planes (indicated by  $(S_C)_1$  and  $(S_C)_2$ ) containing the tool axis and  $V_N$ , which is a normal vector to the position on the desired profile. As defined previously, two contact points,  $(P_{\text{cont}})_1$  and  $(P_{\text{cont}})_2$  can be obtained, and  $(P_{\text{cont}})_1$  moves to  $(P_{\text{cont}})_2$  as the tool turns and follows the path. The movement from  $(P_{\text{cont}})_1$  to  $(P_{\text{cont}})_2$  leads to a trace on the milled surface (cf. Fig. 2). In fact, this trace is the result of the contact points on several planes  $(S_C)_i$  ( $\forall i=1, 2, \dots, N$ ). This trace is inclined at a certain angle. In the case of an ideal milling operation, the inclined angle  $\alpha_M$  can be represented mathematically as

$$\alpha_M = \tan^{-1} \left[ \frac{f \cdot \tan \alpha_{H1}}{2\pi \cdot R \cdot V_R} \right], \quad (3)$$

where  $f$  denotes the feedrate [mm/min],  $\alpha_{H1}$  denotes the helix angle [degree],  $R$  is the tool radius [mm] and  $V_R$  is the spindle speed [r.p.m.]. Equation (3) shows that if  $V_R$  is much bigger than  $f$ ,  $\alpha_M$  converges to zero. In this case, the inclination angle  $\alpha_M$  could be neglected. For example, in our case ( $f=100\text{mm/min}$ ,  $\alpha_{H1}=30^\circ$ ,  $R=3\text{mm}$ ,  $V_R=1250\text{RPM}$ ),  $\alpha_M$  is  $0.14^\circ$ .

On the other hand, the trace of the contact point is complicatedly deformed due to the effects of the tool deflection. Each contact point has a corresponding position with respect to the angular position of the tool. Since the cutting forces vary as a function of the tool angular position, the tool deflection also varies with respect to the vertical position of the contact points. Consequently, the tool deflection can be given by a function of the vertical position of the contact points because it can be represented by the tool angular position. For each contact point, it is possible to integrate the deflection effects. The milled profile can then be obtained by interpolating all the shifted contact points.

Figure 3 shows three different steps for the milled surface generation procedure when the tool is at a certain nominal position. As a first step, all possible contact points  $P_i$  ( $i=1, 2, \dots, N$ ) are considered on the plane  $(S_C)_i$  ( $i=1, 2, \dots, N$ ). Here, the contact points  $P_i$  ( $i=1, 2, \dots, N$ ) stay on the desired profile. In the second step, all tool

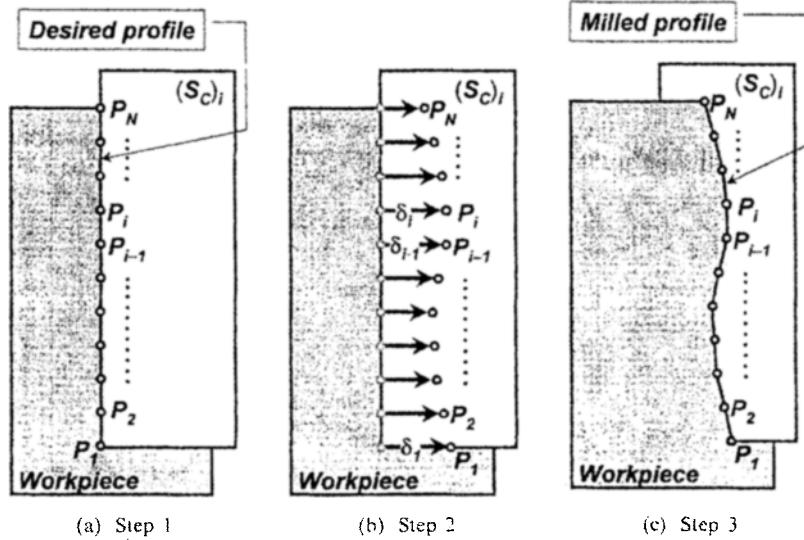


Fig. 3 Generation of milled surface by the contact point

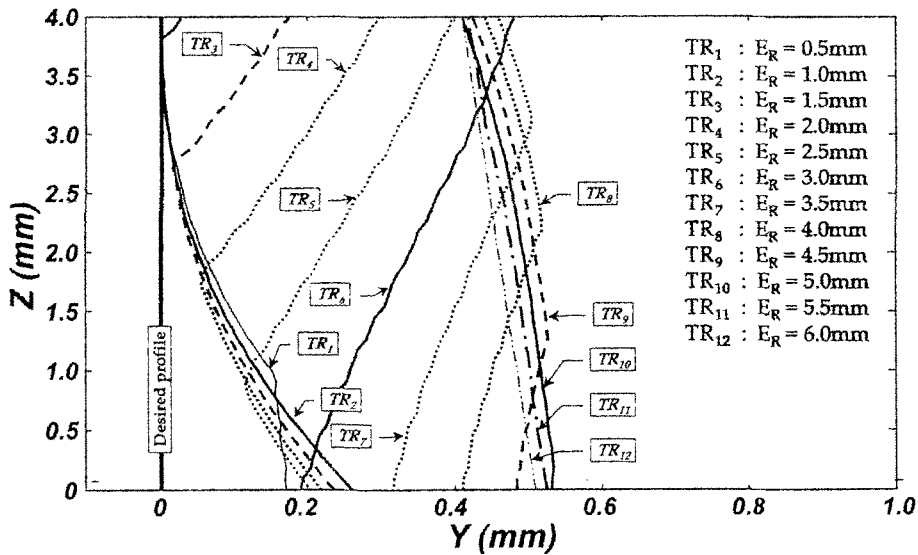


Fig. 4 Simulation of milled surface profiles

deflections are computed with respect to each tool angular position on the plane  $(S_C)_i$  corresponding to the position of the contact point. Finally, a linear interpolation is carried out to obtain the final milled profile. This procedure should be carried out for the entire nominal positions of the tool. Therefore, the set of all obtained milled profiles corresponds to the milled surface generated by the contact point movement.

**2.2 Illustrative examples**

We have performed the required simulations to determine several deformed profiles. The deformed profiles are represented in Fig. 4. In this case, a straight line ( $x=0$ ) corresponds to the desired profile. The deformed profiles are determined when  $A_D=4\text{mm}$ ,  $F_D=0.02\text{mm/tooth}$  and  $R_D$  evolves from 0 to 6mm by increments of 0.5mm. These results show that the traces of the contact points have various geometric forms varying as

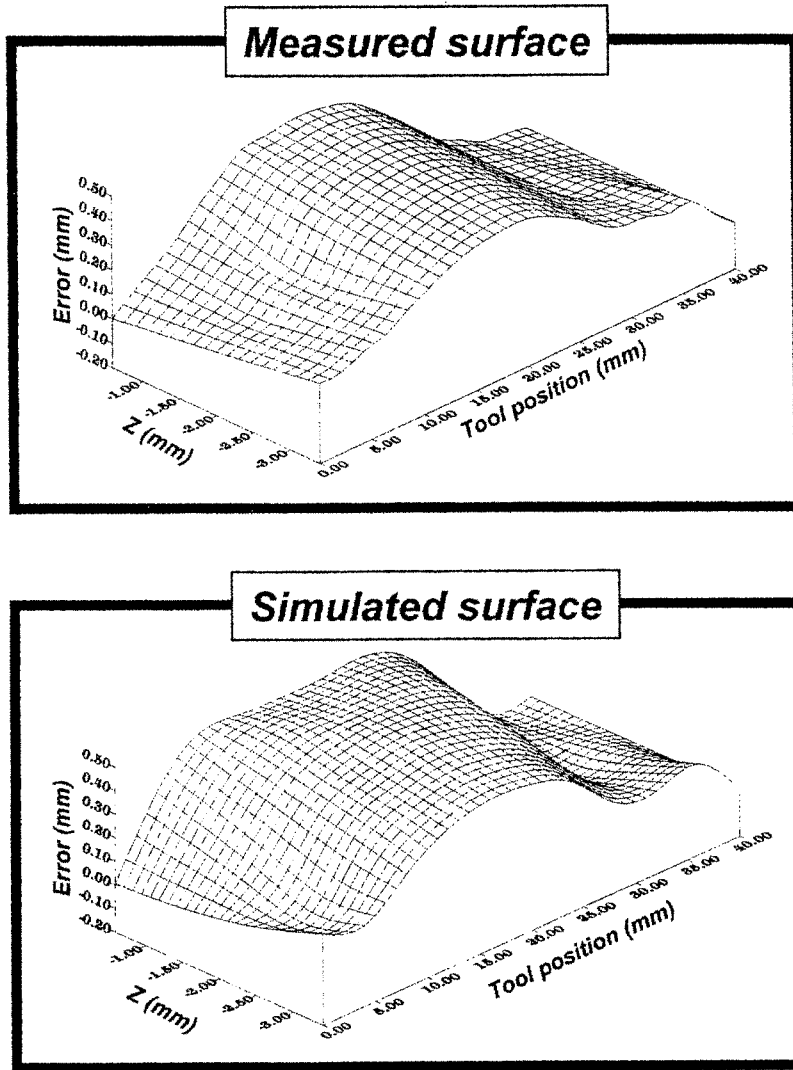


Fig. 5 Measured and Simulated milled surfaces

functions of  $R_D$ .

To consider the error variations along the tool axis, we have compared two typically deformed profiles obtained by applying two different radial depths of cut:  $R_D=6\text{mm}$  (denoted as deformed profile  $TR_{12}$ ) and  $R_D=3\text{mm}$  (denoted as deformed profile  $TR_6$ ). With regard to  $TR_{12}$ , the cutting force is higher than the case of  $TR_6$ . The deviation of  $TR_{12}$  from the desired profile is also larger than that of  $TR_6$ . However, regarding the error interval (difference between the maximum and minimum errors),  $TR_6$  is larger than that of  $TR_{12}$ . The profile  $TR_{12}$  has a 0.1mm difference

between the values at the top ( $z=4\text{mm}$ ) and the bottom ( $z=0\text{mm}$ ), while  $TR_6$  has a 0.3mm difference. Each profile has different directions inclined with respect to the desired profile, which is parallel to the Z-axis.

This fact is very important for the surface error compensation method proposed in Part 1 of this paper. The error interval is much more important than the whole deviation quantity of the milled profile. Since the tool path compensation method allows us to shift the milled profile to the desired profile globally, the narrower the error interval becomes, the greater the possibility that it has to

satisfy the imposed tolerance precisely. This aspect will be more concretely described in the following section.

To illustrate how to establish the surface prediction as the radial depth of cut varies, we have treated the cylindrical workpiece as presented in Part I (cf. Fig. 11 in Part I). Also, a real milling operation has been carried out to validate the simulation results experimentally. Figure 5 shows the measured and simulated surfaces. As shown in the figure, the global form of the simulated surface is very similar to that of the measured surface.

### 3. Definition of Compensation Criteria

In considering the imposed machining tolerances, it is necessary to observe two extremes of the surface errors. These extremes, called the maximal and minimal errors, represent the error interval that should stay in the imposed tolerance interval by the tool path compensation. As a first step, the maximal and minimal errors are defined mathematically.

#### 3.1 Characterization of the surface errors

The maximal error  $E_{max}$  and the minimal error  $E_{min}$  are two extremes on an arbitrary plane ( $S_c$ ), defined as follows:

**Maximal error  $E_{max}$**  : This is the largest algebraic

error on the milled surface with respect to a given coordinate on the desired profile. If this error leads to an undercut with respect to the desired profile,  $E_{max}$  has positive values. Contrarily, if it leads to an overcut,  $E_{max}$  has negative values.

**Minimal error  $E_{min}$**  : This is the smallest algebraic error on the milled surface with respect to a given coordinate on the desired profile. If this error leads to an undercut with respect to desired profile,  $E_{min}$  has positive values. Contrarily, if it leads to an overcut,  $E_{min}$  has negative values.

Figure 6 shows three examples to illustrate the definition of maximal and minimal errors. In Fig. 6(a), the errors  $E_{max}$  and  $E_{min}$  occur at the left side of the desired profile. Since they lead to an undercut,  $E_{max}$  and  $E_{min}$  are positive values. The values of  $E_{max}$  and  $E_{min}$  correspond to 0.25mm and 0.1mm. In Fig. 6(b),  $E_{max}$  occurs at the left side and  $E_{min}$  occurs at the right side of the desired profile. The values of  $E_{max}$  and  $E_{min}$  correspond to 0.2mm and -0.1mm. Here, the values of  $E_{max}$  and  $E_{min}$  correspond to -0.1mm and -0.25mm.

#### 3.2 Compensation reference

To apply the compensation method, it is necessary to choose a reference, called the Compensation Reference. Among infinite possible references, we consider here three typical cases, as shown in Fig. 7. Figure 7(a) schematizes an uncompen-

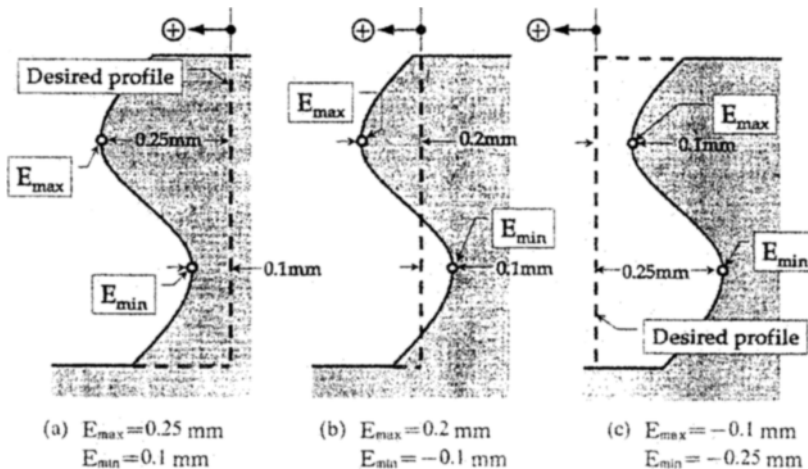


Fig. 6 Examples of the maximal and minimal errors

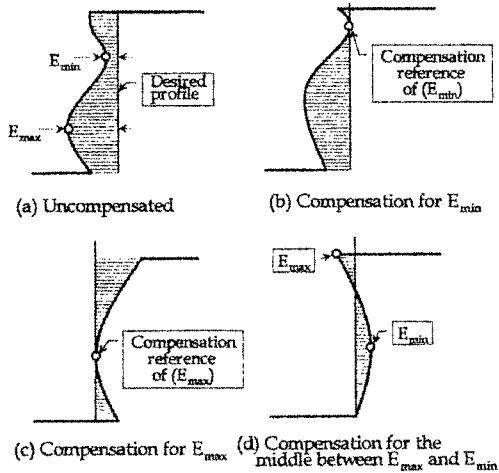


Fig. 7 Compensation references

sated milled profile. As previously mentioned, the profile is given in its complex form. Fig. 7(b) shows the case when  $E_{min}$  is chosen as the compensation reference. Figure 7(c) shows the case when  $E_{max}$  is chosen. In these cases, the tool compensation is carried out such that  $E_{min}$  or  $E_{max}$  should be on the desired profile. However, excessive errors occur at other places. Therefore, in order to reduce all errors, it is necessary to choose another compensation reference.

Figure 7(d) shows the case when the medium value between  $E_{max}$  and  $E_{min}$  is chosen as the compensation reference. In this case, the surface errors are uniformly reduced with an undercut and an overcut. On the other hand, in some cases only an undercut or an overcut is required to satisfy the diverse design concepts. According to the design concept, the desired profile is not always at the center of the given tolerance interval. It is therefore necessary to compensate the errors so that the average value of  $E_{max}$  and  $E_{min}$  should be at the center of the tolerance interval. This solution allows us to effectively reduce the surface errors in the vicinity of the tolerance interval.

### 3.3 Cutting process simulation

Before executing the tool path compensation, some aspects should be verified. When the uncompensated milled surface stays inside the tolerance interval, it is unnecessary to carry out the

tool path compensation. If the uncompensated milled surface does not stay inside the tolerance interval, the tool path compensation is required. Moreover, there exist some cases that the compensated milled surface cannot stay inside the tolerance interval any longer, in spite of making an effort to execute the tool path compensation. For these aspects, a simulation is required to do a series of verifications before choosing a solution. We present here a cutting process simulation based on the surface prediction as previously proposed.

#### 3.3.1 Fulfillment of the manufacturing tolerances

As a first step, we investigate the fulfillment of a given tolerance  $N_{c_{min}}^{c_{max}}$ , where  $c_{max}$  and  $c_{min}$  are the tolerance limits. When we manufacture a workpiece by the nominal tool trajectory, if tolerance  $N_{c_{min}}^{c_{max}}$  is given, we can define two parameters  $\Delta_{max}$  and  $\Delta_{min}$  as follows:

$$\begin{aligned} \Delta_{max} &= c_{max} - E_{max} \\ \Delta_{min} &= c_{min} - E_{min} \end{aligned} \quad (4)$$

By considering these parameters, we can verify whether or not the given tolerance  $N_{c_{min}}^{c_{max}}$  satisfies the following condition. In spite of the spread errors on the milled surface, if they satisfy the tolerance criteria, the milling operation can be approved.

**Condition 1** : Verification of tolerance fulfillment.

● If  $\Delta_{max} > 0$  and  $\Delta_{min} < 0$ , the tolerance  $N_{c_{min}}^{c_{max}}$  is satisfied. There is no need to compensate.

If condition 1 is not satisfied, we need to compensate the tool path. Therefore, it is necessary to verify the possibility of compensation according to the following conditions.

#### 3.3.2 Possibility of the tool path compensation

If the given tolerance is not satisfied, it is necessary to compensate the errors through tool path modification. Before applying the compensation method, the verification procedure is mandatory in order to check whether it is possible to

satisfy the imposed tolerance by error compensation. This allows us to avoid the unnecessary compensation process. However, it is not as simple as the condition 1 presented previously. Since the tool path compensation leads to the change in cutting conditions, the same surface form previously used cannot be kept. It is necessary to observe the surface form for all possible cutting conditions. Thus, by characterizing the surface form using the error interval, we can suggest a condition to verify the possibility of the tool path compensation.

The maximal error interval  $(IE)_{max}$  is defined as the maximal value among the whole error interval IE for all the possible cutting conditions to be met in the milling operation. If the maximal error interval  $(IE)_{max}$  is narrower than the tolerance interval IT, it is possible to compensate the errors inside the tolerance criteria. Although the error interval IE varies by the tool path compensation, IE enters inside the tolerance criteria and does not exceed IT if  $(IE)_{max} < IT$ .

**Condition 2 :** Verification of the possibility of tool path compensation.

- If  $\{IT - (IE)_{max}\} < 0$ , it is possible to com-

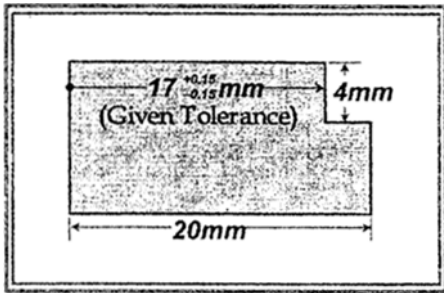


Fig. 8 Desired surface and given tolerance

pensate the errors inside the tolerance criteria.

## 4. Experimental Examples

### 4.1 Linear profile milling operation

In this study, we have treated a linear profile milling operation as a practical example to illustrate the previously proposed approaches. Fig. 8 shows the geometric form of this example. In this case, the imposed tolerance range is  $17^{+0.15}_{-0.15}$ , and a flat-end milling tool (4 flutes, 6mm diameter, 30°-helix angle, 30mm-used length) is used to machine a steel workpiece (middle carbon steel). The spindle speed is fixed to 1250 r.p.m., and the cutting conditions correspond to the following formula:  $R_D=3mm$ ,  $A_D=4mm$ ,  $F_D=0.02mm/tooth$ . To show the effectiveness of our research, both uncompensated and compensated milling operation simulations are carried out.

#### 4.1.1 Uncompensated milling operation

The uncompensated milling operation is presented in Fig. 9. After the cutting process simulation, we have obtained the uncompensated milled profile. By computing the parameters  $\Delta_{max}$  and  $\Delta_{min}$ , both conditions (condition 1 and 2) can be applied to verify the fulfillment of the given tolerance and the possibility of compensation.

As illustrated in Fig. 9, the milled surface stays outside the tolerance criteria. As a result, we cannot obtain the milled surface satisfying the tolerance. In fact, it can be verified by condition 1. However, we can verify the possibility of compensation by condition 2.

#### 4.1.2 Compensated milling operation

The compensated milling operation is present-

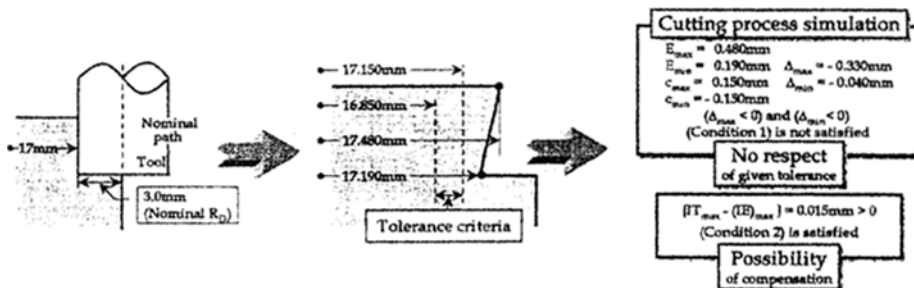


Fig. 9 Cutting process simulation for uncompen-sated milling operation



ed in Fig. 10. As verified above by condition 2, we have obtained a compensated tool path by the compensation method as presented in Part I. The average of  $E_{max}$  and  $E_{min}$  is taken as the compensation reference. After milling the surface with the compensated radial depth of cut, we have obtained a milled surface that stays inside the tolerance criteria. As a result, we have succeeded in obtaining the milled surface satisfying the imposed tolerance.

**4.1.3 Experimental validation**

In order to validate these simulation results, we have conducted an experiment with the same cutting conditions as in the previous example. Figure 11 shows two cases of milled surfaces: (1) an uncompensated milled surface and (2) a compensated milled surface. The uncompensated surface is outside the given tolerance criteria, but the milled surface with compensation is inside the criteria. It is also verified that the experimental

results correspond to those of the simulation.

**4.2 B-spline profile milling operation**

The previous example has not dealt with a case of the cutting conditions varying along the cutter path. In general flat-end milling, the cutter cannot avoid meeting the varied cutting conditions. The radial depth of cut varies notably along the tool path, although the constant thickness remains all around the desired profile after rough cutting. Therefore, this second example corresponds to a profile milling operation, which is designed as a B-spline curve (cf. Fig. 12).

Two milling operations (rough and finish cuttings) are required to manufacture the desired workpiece. The rough cutting has been carried out to maintain 3mm of constant thickness all around the desired profile. However, the radial depth of cut varies during finishing regardless of the constant thickness. The axial depth of cut consistently corresponds to 4mm. The finishing is

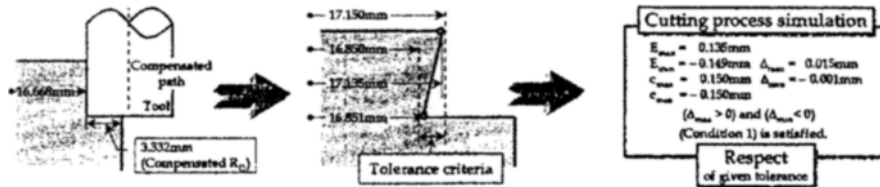
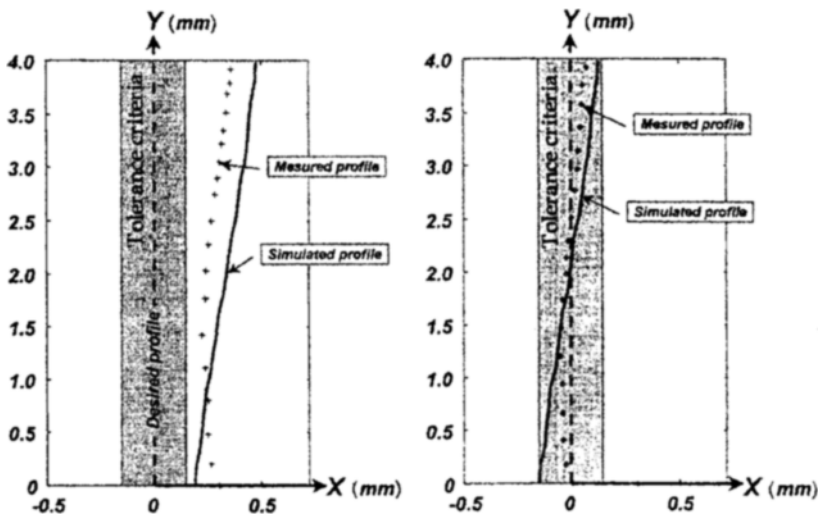


Fig. 10 Cutting process simulation for compensated milling operation



(a) Uncompensated profiles (b) Compensated profiles

Fig. 11 Experimental results

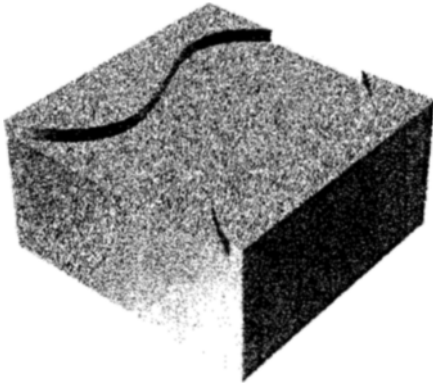


Fig. 12 Geometry of the desired workpiece

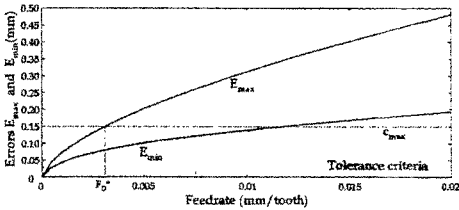


Fig. 13 Variation of  $E_{max}$  and  $E_{min}$  with respect to feedrate

carried out under the same machining conditions as those used in the previous example. The manufacturing tolerance is given as  $\pm 0.15$ mm all around the desired profile.

In order to emphasize the effectiveness of the proposed approaches, we can take into account a solution often used in the industry. This solution consists of reducing the feedrate until the tolerance is fulfilled. Therefore, it is necessary to determine a critical value of feedrate that leads to smaller errors, which can be inside the tolerance criteria. To determine this value, we must consider that  $E_{max}$  and  $E_{min}$  vary according to the feedrate. Figure 13 shows the variation of  $E_{max}$  and  $E_{min}$  from 0.0 to 0.02 [mm/tooth]. As presented in Fig. 13, we can obtain a critical value of the feedrate (indicated by  $F_d^*$ ). In order for  $E_{max}$  to be inferior to 0.15mm of the maximum condition of tolerance, the critical feedrate  $F_d^*$  must be smaller than 0.003mm/tooth. When the milling operation is executed with this critical feedrate, it is possible to fulfill the tolerance; however, in this case, the productivity is reduced by 85%. While considering this solution, we can

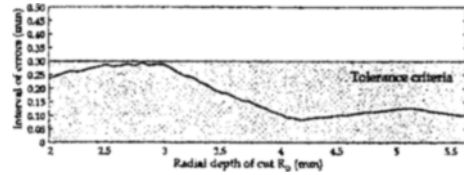


Fig. 14 Error interval with respect to radial depth of cut

anticipate the unavoidable damages on productivity. Therefore, the importance of the compensation process is emphasized.

#### 4.2.1 Cutting process simulation

In the cutting process simulation, it is possible to observe that the majority of the milled surface errors are outside the tolerance interval. This means that Condition 1 is not satisfied. It is obvious that the given tolerance is not fulfilled on the global milled surface of the manufactured workpiece. In this case, it is necessary to correct the tool path. We verify if it is possible to compensate the errors by checking Condition 2. Figure 14 shows a variation of the error interval between 2mm and 5.5mm. This corresponds to the whole domain of radial depth of cut possibly encountered during the milling operation. This error interval always stays inside the tolerance interval. Therefore, it is possible to fulfill the tolerance by correcting the tool path.

#### 4.2.2 Experimental validation

We have carried out two milling operations with the nominal and the corrected tool paths to validate our simulation results. First, we have manufactured two identical roughed workpieces. Subsequently, two finish cuttings are carried out – uncompensated cutting and compensated cutting. After these operations, we have obtained coordinate information about the milled surface by using a Coordinate Measuring Machine (Zeiss, MC550) (cf. Fig. 15).

Figure 16(a) shows the milled surface of the workpiece manufactured with the nominal path (uncompensated). The four curves correspond to the surface errors at four different altitudes. The majority of these curves are outside the tolerance criteria. Therefore, in this case, the given toleran-

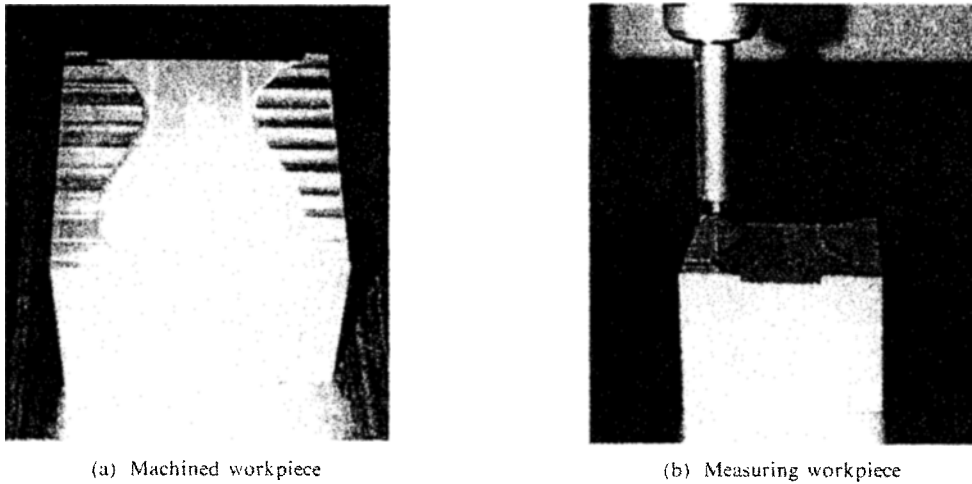


Fig. 15 Experimental works

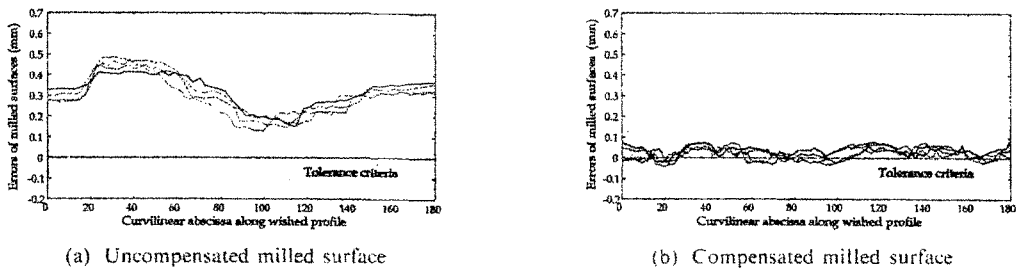


Fig. 16 Measured error distribution of milled surface

ce is not satisfied. Figure 16(b) shows the four measured curves on the milled surface manufactured with the corrected path. All the surface errors remain inside the tolerance interval. The amplitude of the surface errors of the compensated milled surface is smaller and more stable than that of the uncompensated milled surface. We succeeded in compensating the errors with respect to the given tolerance, as predicted in the simulation.

## 5. Discussion

If the given tolerance criteria is very tight (very narrow allowable error interval), the path compensation method may not be effective in a real situation. In this case, we can consider other approaches to improve the surface quality. The first possible solution can be the optimization of the cutting conditions. For example, it is possible to control the feedrate actively so that the errors

cannot exceed the tolerance criteria during the whole milling operation. When rough cutting is required, it is possible to optimize the depth of cut. Since the proposed cutting process simulation allows us to predict the resultant milled surface in any case, we can generate the tool path for rough cutting so that the errors do not exceed the tolerance criteria when finish cutting is executed. Moreover, if these optimizations are combined with the tool path compensation, it is possible to maximize the compensation performance. Also, if it is possible to redefine the tolerance based on the concept of *Design for Manufacturing* (Rembold et al., 1993), the cutting process simulation can provide us with a new tolerance criteria that can be easily satisfied.

## 6. Conclusion

Following Part I, which proposed the tool path compensation methodology, we have presented

concrete application methodologies for the compensation.

(1) To analyze the tool deflection effect on the milled surfaces precisely, we have presented a model of surface prediction. This model is based on the behavior of the significant contact points, which mainly contribute to generate the milled surface. These significant contact points are affected by the tool deflection during the milling operation. We have analytically illustrated how they lead to the surface errors due to the deflection effects, and proposed a concrete procedure for the surface form generation. To represent it, we have carried out cutting process simulations under several different cutting conditions. From the simulation results, we can obtain important matters for the tool path compensation. The surfaces affected by the deflection effects appear as complicated shapes since the diverse errors are spread on the whole surface. If a manufacturing tolerance is given, it is possible to compare two extremes of the surface errors with it. The compensation method requires choosing a certain reference to shift the surface globally closer to the desired surface. This means that it may be possible to put the entire surface inside the tolerance criteria if a reasonable choice of the compensation reference is made.

(2) Before compensating the tool path it is necessary to verify whether or not the tolerance is satisfied during the uncompensated milling operation. Although the compensation is required, it is also necessary to verify whether the compensation can make the surface to satisfy the imposed tolerance or not. These prerequisite verifications allow us to avoid unnecessary compensation. Thus, we have presented two concrete conditions used to verify these matters based on the surface prediction.

(3) To illustrate the proposed approaches in this paper, we have presented practical examples. First, we have simulated two milling operations, uncompensated and compensated. By applying the cutting process simulation, it has been verified that the uncompensated surface does not satisfy the tolerance criteria, and it is possible to shift the errors inside the tolerance interval by the path

compensation. Next, in order to verify the simulation results, required experiments have been performed under the same conditions, and we have obtained reasonable results with good agreements.

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